



# An algorithm for the search of homogeneous strain-rate fields

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## What's new

Starting from a Fortran code for the estimation of 3D strain-rate tensor under the hypothesis of homogeneous strain behavior, not requiring a priori discretization of the network in polygons or triangles (Pietrantonio & Riguzzi, *Journal of Geodynamics*, 2004), we implemented a new algorithm for the automatic search of sites belonging to an initial user-defined homogeneous strain-rate field.

The new software, developed in Matlab, is able to find areas of homogeneous strain behavior, taking into account all the available 3D GPS velocities in the area and performing suitable statistical tests.

## The statistical tests

Both the statistical tests are based on the standard hypothesis of normal observations and are carried out at the chosen confidence level.

### Global test

The aim is to verify if the model (functional + stochastic) is globally correct (homogeneity of the strain-rate field + absence of outliers)

$$H_0 : \hat{\sigma}_0^2 = \sigma_0^2$$

where  $\hat{\sigma}_0^2$  = posterior variance factor.

The test statistics is a  $\chi^2$  with  $r$  degrees of freedom ( $r$  = redundancy)

$$\chi_{\text{exp}}^2 = r \frac{\hat{\sigma}_0^2}{\sigma_0^2}$$

If  $\chi_{\text{exp}}^2 < \chi_r^2$  the model is correct.

### Local tests

The aim is the outlier detection

$$H_0 : \hat{v}_k = 0$$

where  $\hat{v}_k$  = residual of the  $k$ -th observation.

The test statistics (normalized residual) for the  $k$ -th observation is a Thompson variate with  $r$  degrees of freedom

$$\tau_{k,\text{exp}} = \frac{e_k^T P \hat{v}}{\hat{\sigma}_0 \sqrt{e_k^T P Q_{vv} P e_k}}$$

where  $P$  = weight matrix of input velocities,  $Q_{vv}$  = cofactor matrix of residuals,  $e_k$  = null vector except for the  $k$ -th element equal to 1.

If  $|\tau_{k,\text{exp}}| > \tau_r$  the  $k$ -th observation is a possible outlier.

## The algorithm

### 1. User definition of the initial field

The strain-rate tensor is least-squares estimated starting from the hypothesis of homogeneous strain-rate field.

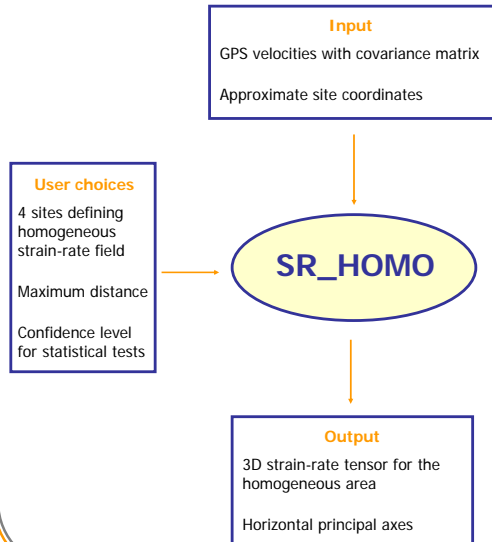
The choice of the initial 4 sites defining an homogeneous strain-rate field ("seed" sites) is iteratively performed by the user, on the basis of the result of the  $\chi^2$  test on the homogeneity of the field.

### 2. Adding sites...

Once an initial homogeneous strain-rate field is defined, the software automatically searches for sites belonging to this field, starting from the site nearest to the barycentre of the "seed" sites and proceeding until a user-defined maximum distance.

The definition of the area of homogeneous strain-rate is driven by the results of suitably implemented statistical tests: each time a site is added, the strain-rate tensor is re-estimated and both  $\chi^2$  global test and Thompson local tests are performed to verify the homogeneity of the newly obtained field and the eventual presence of outliers.

## Sketch of the software



Choices	
"Seed" sites	MPRA UDIN ACOM KOET
Maximum distance	150 km
Confidence level	99%

GPS velocities are expressed in an Eurasia-fixed reference frame.

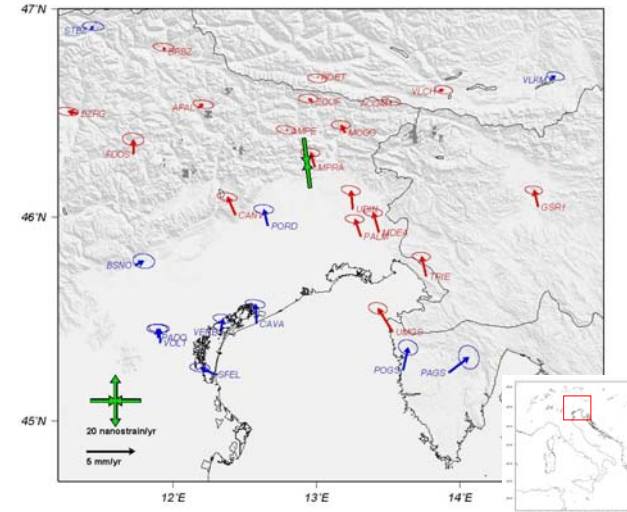
An area of more than 100 km<sup>2</sup> of homogeneous strain-rate is detected, inside the whole considered area shown in the figure.

In red the accepted sites, in blue the rejected ones.

Site PORD is rejected, since both global and local tests fails, because of problems in the height component (sw works in 3D...)

## Test case: NE of Italy and around

- Alpine compression, about N-S:  $(-20 \pm 3) \cdot 10^{-9} \text{ yr}^{-1}$
- Non significant extension:  $(1 \pm 2) \cdot 10^{-9} \text{ yr}^{-1}$



Global test	
$\chi_{\text{exp}}^2$	77.4
$\chi_r^2$	71.3

Local tests	
Sites	$\tau_{\text{exp}}$
MPRA	-2.023
UDIN	-0.342
ACOM	-0.342
KOET	-1.593
MOGG	-0.114
ZOUF	-0.267
AMPE	-0.028
PALM	-0.234
MIDA	-1.313
VLCH	-0.140
CANV	-0.188
AFAL	-0.274
TRIE	-0.232
BBRZ	-0.201
UMGS	-0.201
OSRI	-0.243
FDOS	-1.231
BZRG	-0.174
	-1.289
	-0.195
	-0.125
	-0.251
	-2.365
	-0.353
	-1.027
	-1.148
	-0.817
	-0.022
	-1.143
	-0.500
	-0.708
	-0.101
	-0.830
	-0.350
	-0.622
	-0.240
	-0.681
	-1.565
	-1.741
	-0.820
	-1.223
	-0.844
	-0.226
	-0.306
	-2.145
	-0.084
	-0.190
	-0.700
	-0.974
	-0.472